In its article on temperature, Encarta says: “In the case of two bodies at different temperatures, heat will flow from the hotter to the colder until their temperatures are identical and thermal equilibrium is reached.” A lot the forces in nature work this way. Nature’s goal oftentimes is to establish equilibrium. In this lab, we will experiment with one of those forces, heat transfer, and see how it works.

Equipment:

- A thermometer. We will loan you one. We want them all back when you are done.
- A way of heating water to around 50°C (=112°F). A saucepan on a stove works well.
- At least two different containers to place the liquid in to cool. The saucepan itself works well for one of them. It is fun if one of them is well insulated, like using a Styrofoam cup or wrapping towels around the saucepan.
- Notebook paper on which to write down the temperatures of the liquid every minute for an hour.
- Excel, CricketGraph, and word processing (either ClarisWorks or Word) software.

Data Collection Procedure:

1. Record the temperature in whatever environment is surrounding the container. This is called the ambient temperature, or $T_A$. Run experiments that use at least two different ambient temperatures. You could possibly allow the liquid to cool outdoors or in a refrigerator instead of indoors.

2. Heat the liquid in the saucepan to 50°C. Heat it slowly so that you don’t overshoot 50°C. Stir it constantly to be sure that the liquid is all the same temperature. Be careful! While 50°C is well short of boiling, it is still hot enough to burn you.

3. Pour the liquid into whatever container you are using for this experimental run. Place the thermometer into the liquid. Record the starting temperature. This is called the original temperature, or $T_o$.

4. Every minute, observe the thermometer and record the temperature of the liquid as it cools. Do this for one hour. Stir the liquid gently to be sure that it is all the same temperature. Even though the °C marks are 2°F apart on the thermometer, try to record the temperature to the nearest degree.
5. When you are done with the experiment, enter the time (in minutes from the start of the experiment) in the first column of an Excel spreadsheet. Enter the temperature (in °C) in the second column.

6. Copy and paste the time and temperature spreadsheet columns into CricketGraph.

7. Create a graph of the temperature (y) versus time (x). Experiment with different curve fits. (But, be sure to check out the comment at the end of this write-up before taking any curve fits too seriously.)

8. Compute the cooling factor C for each experiment. To determine C for an experiment, divide 0.693 by the number of minutes it takes for the liquid to cool to a temperature halfway between the original temperature and the ambient temperature. The cooling factor will be explained in more detail below.

**Turn in on January 4:**

- The thermometer.
- A report describing what you did, what containers and ambient temperatures you used, what numbers you collected, and the graphs that resulted. Copy and paste the numbers from the Excel spreadsheets and the graphs from CricketGraph right into your word processor report.

**How it Really Works – Calculus meets Physics:**

In most of the math we learn in high school, things have equations for what they are. For example, the equation for a straight line on a graph is \( y = mx + b \). If we know x, then we know what y is.

In the world of physics, a lot of things do not have equations for what they are, but for how they are changing. This is because nature likes to put things in equilibrium. In your temperature experiment, the forces of nature will work towards making the temperature of the liquid the same as the temperature of the surrounding air. We call this surrounding air the ambient temperature.

When you take calculus, you will learn to write a heat transfer equation that looks like this:

\[
\frac{\Delta T}{\Delta t} \propto T_A - T
\]

In English this says that “the change of the liquid’s Temperature during a change in time is proportional to the difference between the ambient temperature and the liquid’s temperature right now.” This makes sense. The bigger the difference, the faster the rate at which nature will try to cool it off. (This is one of the nice things about calculus. The math may be harder, but understanding what is going on doesn’t have to be.)
So, we start out with an equation for how the temperature changes and then we use calculus to solve for what the temperature will be as the liquid cools. When you do this, you get:

$$T = T_A + (T_0 - T_A) e^{-Ct}$$
In this equation:

\[ \text{t is the time in minutes that the liquid has been cooling} \]
\[ \text{T is the temperature of the liquid in } ^\circ\text{C at time } t \]
\[ \text{T_A is the ambient temperature in } ^\circ\text{C} \]
\[ \text{T_o is the original temperature of the liquid in } ^\circ\text{C (this should be approximately 50}^\circ\text{C)} \]

\( e \) is a special number in mathematics. It is called the \textit{exponential}. Its approximate value is 2.71828. Like \( \pi \), it actually has an infinite number of decimal digits. On scientific calculators, you can take \( e \) to the \( x \) power with a key that usually looks like \( e^x \). In spreadsheets, you take \( e \) to the \( x \) power with the expression \( \text{exp}(x) \).

\( C \) is called the \textit{cooling factor}. It is a special number that tells you how fast the liquid cools. \( C \) depends on what liquid you are using and on how well the container is insulated. Heat transfer scientists spend lots of time determining the different \( C \) values for different materials and containers so that they have an equation to predict heat transfer behaviors in the future. (For example, will a truckload of frozen strawberries reach the market before they thaw? What if the driver drives at night instead of the hot day?)

To determine \( C \) for your experiments, divide 0.693 by the number of minutes it takes for the liquid to cool to a temperature halfway between the original temperature and the ambient temperature. This now gives you an equation to predict how the liquid in this container will cool.

You can put your predictions right into your spreadsheet. For example, suppose your original temperature is 50\(^\circ\text{C}\) and the ambient temperature is 22\(^\circ\text{C}\). If you determine that \( C \) for this experiment is 0.028 and you have the times in column A starting with cell A1 and the recorded temperatures in column B starting with cell B1, you can place the theoretical temperature in cell C1 with the following equation:

\[ = 22 + (50-22) \times \text{exp}(-0.028 \times A1) \]

You can also check how close your measurements were to the theoretical temperatures by placing in cell D1 the equation:

\[ = B1 - C1 \]

You can then copy and paste these formulas into the other rows in columns C and D.

**Things to Notice:**

- Some of the curve fits in CricketGraph may seem to fit your data quite well, but in fact none of the curve fitting equations available in CricketGraph matches the form that we know from physics that the equation must take. Thus, if you were to blindly use one of these curve fitting equations to predict how the liquid will continue cooling, you would get the wrong answer. **This is why the blind use of computers does not replace a science and math education!** You must understand the science and math to properly direct the computer to a
correct solution. The computer is a *tool* for scientists, not a replacement for them.

- Nature tries to bring things into equilibrium faster when they are more out of balance. This is why your liquid cools quickly at first and then cools slower. This is also why liquids cool faster when the difference between the original and ambient temperatures is greater.

- Notice how the cooling factor \( C \) varies between different containers. Larger cooling factors represent faster cooling, smaller ones represent slower cooling.

**Example Graph:**

When we ran this experiment at home, we got a graph like this one.

**Cooling of Hot Water**

For this experiment, the ambient temperature, \( T_A \), was 22°C. The original temperature, \( T_0 \), was 50°C.

Halfway between \( T_A \) and \( T_0 \) is 36°. We found that it took 25 minutes to reach 36°, so the cooling factor for this experiment was \( \frac{0.693}{25} = 0.028 \).

So the temperature of this cooling liquid has an equation of:

\[
T = 22 + 28 e^{-0.028t}
\]
Or, in Excel:

\[
= 22 + 28 \exp(-0.028A1)
\]

This experiment was only run for 40 minutes. Be sure to run your experiments for the full hour to get a better idea of the shape of the curve.